Effect of Hypersonic Nonlinear Aerodynamic Loading on Panel Flutter

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The system considered is a two-dimensional isotropic panel, or plate-column, on hinged supports, with one end spring restrained in the plane of the panel. Panel geometric nonlinearities and piston-theory aerodynamic nonlinearities are included. Results from an earlier preliminary study indicate that only two second-order nonlinear aerodynamic terms are important. The nonlinear aerodynamic terms introduce the possibility of an amplitude-sensitive instability, where the panel is unstable to disturbances of a certain magnitude but stable for smaller ones. This type of instability is examined for various panel in-plane loads and initial conditions, with other parameters having values representative of current practice. A single new interaction parameter, representing the importance of the nonlinear aerodynamic terms in comparison with the panel geometric nonlinear terms, is introduced. Parameter surveys show that increasing this interaction parameter reduces monotonically the dynamic pressure at flutter, and that the static-pressure difference has a relatively minor effect, with increased cavity pressure stabilizing and decreased cavity pressure destabilizing. On the other hand, the variation of the flutter dynamic pressure with initial energy level and with initial deformation is very complicated. While it is difficult to generalize from such a simple model, the results do suggest that an accurate determination of the in-plane edge restraint is crucial to any assessment of the likelihood of amplitude-sensitive instability.

Nomenclature

a = panel chord
a_k = dimensionless modal amplitude:

$$w(x,t) = [h/(\alpha)^{1/2}] \sum_{k=1}^{N} a_k(t) \sin k\pi \frac{x}{a}$$

$\mathbf{A}_0$ = vector of initial modal amplitudes and velocities $[a_1(0),\ldots,a_N(0), \dot{a}_1(0),\ldots,\dot{a}_N(0)]$
D = plate modulus, $Eh^3/12(1-\nu^2)$
E = Young's modulus
E_0 = dimensionless initial energy, see Eq. (3)
h = panel thickness
K = running spring constant for panel in-plane restraint spring
M = Mach number
N = number of modes used to represent $w(x,t)$
p = pressure
p_∞ = freestream pressure
Δp = dimensionless static pressure difference across panel, $\bar{\Delta} p a^4(\alpha)^{1/2}/Dh$ (positive if cavity pressure exceeds freestream pressure)
q = freestream dynamic pressure
R_x = dimensionless in-plane applied load, $\bar{R}_x a^2/D$ (positive for tension)
t = time
w = panel transverse displacement
x = panel chordwise coordinate
α = dimensionless in-plane restraint parameter, $K[K+Eh/a(1-\nu^2)]^{-1}$
γ = gas constant (here 1.4)
λ = dimensionless dynamic-pressure parameter, $2qa^3/MD$
μ = mass ratio, $\rho_\infty a/\rho h$
ν = Poisson's ratio
ρ = panel mass density
ρ_∞ = freestream mass density
τ = dimensionless time, $t(D/\rho h a^4)^{1/2}$
$(\dot{\ })$ = derivative with respect to τ
$(\bar{\ })$ = dimensional quantity

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Introduction

FOR some time now it has generally been accepted that an important part of the phenomenon of panel flutter is the nonlinear effect resulting from the interaction between transverse panel displacement and in-plane stretching. A recent survey article[1] on panel flutter has discussed the theoretical treatment of this type of nonlinearity in some detail. It generally has a stabilizing effect on flutter, since increasing transverse displacement is resisted by the in-plane stretching. At hypersonic speeds, another nonlinear effect enters the picture. Here, the nature of the aerodynamic loading is such that it too must be regarded as nonlinearly related to the panel motion. Furthermore, the nonlinear aerodynamic loading has a fundamentally different character, in that it reinforces, rather than resists, increasing panel displacement. When this happens, two effects are observed (at least theoretically): the postcritical response of the panel is altered, and in more extreme cases the panel is unstable to large disturbances but stable for small ones. The object of this paper is to investigate further certain aspects of the latter effect, the amplitude-sensitive instability.

Such a problem can be approached in two ways. First, it can be viewed as a specific example of a general class of stability problems involving nonlinear nonconservative systems. There is a substantial body of literature in this area, although to the author's knowledge only one paper[2] has appeared within the last few years that deals specifically with amplitude-sensitive instabilities; it proposes a stability criterion based on the energy in the initial disturbance applied to the system. Other methods, such as Lyapunov's method or the harmonic-balance method, are also capable of predicting this type of instability. However, their application is increasingly difficult as the number of unknowns (modal amplitudes for panels, say) becomes larger. Librescu[3] used the method of Lyapunov in his study, and Bolotin et al.[4] applied the harmonic-balance method in theirs. The latter authors also confirmed their results by direct integration with time of the differential equations for the modal amplitudes. This technique is also used here; it has been applied previously in this country to panel-flutter problems by Dowell.[1] A second viewpoint is to deal directly with the panel-flutter amplitude-sensitive instability in a more pragmatic manner, as in Refs. 3 and 4 cited above. This paper reflects the more pragmatic approach and will present some results for realistic parameter

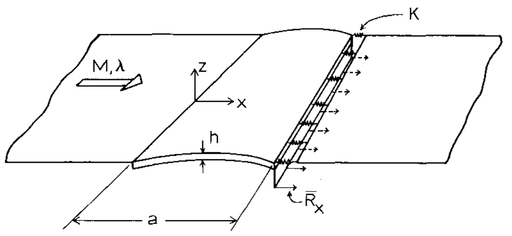

Fig. 1 Two-dimensional panel (plate-column) on hinged supports.

ranges and realistic aerodynamic flowfields. A preliminary examination, reported in an earlier paper,[5] is extended to include the effects of variations in system parameters on an amplitude-sensitive stability boundary.

Equations of Motion

The panel to be considered is illustrated in Fig. 1. It is an isotropic panel, or plate-column, of infinite span. The edges are pinned, and at one end there are a linear in-plane restraint spring and an applied in-plane load. The derivation of the equations of motion is discussed in detail in Ref. 5. Panel in-plane and transverse displacements are approximated as a finite series of assumed modes satisfying the pinned-edge boundary conditions. Equations of motion are obtained from Hamilton's principle, where the expression for the strain energy in the panel is the one that leads to the familiar von Kármán structural operator. The transverse aerodynamic loading is obtained from second-order piston theory, along with a term to account for static pressure differences between the panel cavity and the free stream. As noted in Ref. 5, there are only two nonlinear aerodynamic terms that prove to be important for the parameter ranges considered—one proportional to $(\partial w/\partial x)^2$, and one proportional to $(\partial w/\partial x)(\partial w/\partial t)$. (Symbols are defined in the Nomenclature.) Finally, the in-plane motion equations are used to eliminate all unknowns except the modal amplitudes for the transverse displacement, and the resulting set of equations is

$$\frac{1}{2}\ddot{a}_k+\frac{1}{2}\left(\lambda\frac{\mu}{M}\right)^{1/2}\dot{a}_k+\frac{\pi^2k^2}{2}[R_x+\pi^2k^2]a_k+\tfrac{3}{2}\pi^4k^2a_k\sum_{l=1}^{N}l^2a_l{}^2+$$
$$\lambda\sum_{l=1}^{N}\frac{kl[1-(-1)^{k+l}]}{k^2-l^2}a_l-\frac{[1-(-1)^k]}{k\pi}\Delta p+\frac{(\gamma+1)\pi k}{4}\times$$
$$\left[\frac{\lambda Mh}{a(\alpha)^{1/2}}\right]\sum_{l,m=1}^{N}\frac{lm(k^2-l^2-m^2)[1-(-1)^{k+l+m}]}{[k^2-(l-m)^2][k^2-(l+m)^2]}a_la_m+$$
$$\frac{\pi(\gamma+1)}{8}\left[\frac{Mh}{a(\alpha)^{1/2}}\right]\left(\lambda\frac{\mu}{M}\right)^{1/2}\times$$
$$\left[\sum_{\substack{l,m=1\\l+m=k}}^{N}la_l\dot{a}_m+\sum_{\substack{l,m=1\\l-m=k}}^{N}(ma_m\dot{a}_l-la_l\dot{a}_m)\right]=0,\quad k=1,2,\ldots,N \qquad (1)$$

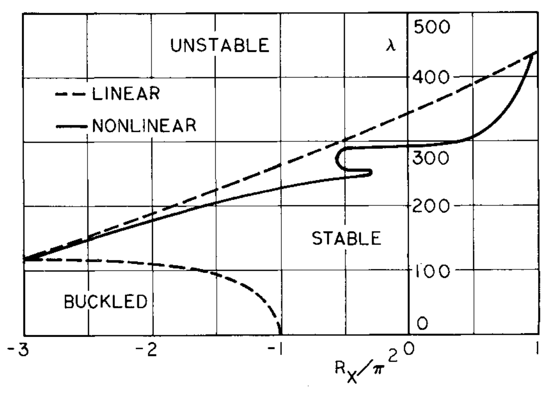

Fig. 2 Comparison of linear and nonlinear stability boundaries (after Ref. 5). $N=6$, $\mu/M=0.01$, $Mh/a(\alpha)^{1/2}=0.158$, $\Delta p=0$, $E_0=175$ with $a_1(0)=-a_2(0)\neq 0$.

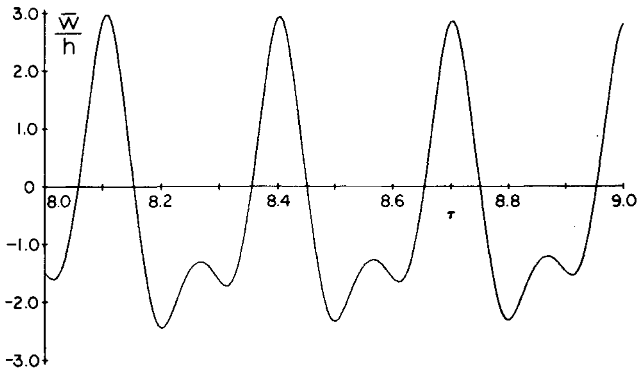

Fig. 3 Dimensionless panel displacement at $x/a=0.75$ vs dimensionless time. $N=6$, $R_x=-\pi^2$, $\lambda=260$, $\mu/M=0.01$, $Mh/a(\alpha)^{1/2}=0.158$, $\Delta p=0$.

The notation used here is slightly different from that used in Ref. 5, most notably in the definitions of dimensionless dynamic pressure and dimensionless in-plane applied load. Also, a term is included here for the static pressure difference. Note now that the panel motion can be represented as follows:

$$w(x,t)=[h/(\alpha)^{1/2}]Fn[x/a,\tau;\mathbf{A}_0,R_x,\lambda,\mu/M,\gamma,Mh/a(\alpha)^{1/2},\Delta p] \qquad (2)$$

Thus, even in the presence of nonlinear aerodynamic loading of the type considered here, the parameters can be arranged so that the in-plane restraint parameter α appears only in conjunction with the aerodynamic parameter Mh/a. Amplitude-sensitive instability occurs where the asymptotic panel motion is dependent on the initial conditions, as represented by the initial state vector $\mathbf{A}_0$.[5]

Initial conditions will be determined by fixing the magnitude of the initial energy in the panel. An expression for this energy is derived in Ref. 5; it can also be written so as to eliminate explicit dependence on α:

$$\bar{E}_0=(Dh^2/a^3\alpha)E_0=(Dh^2/a^3\alpha)Fn[\mathbf{A}_0,R_x] \qquad (3)$$

The panel motion is determined by integrating Eqs. (1) with time from given initial conditions $\mathbf{A}_0$. The integration is carried out over a long enough interval of time to determine whether or not the motion is stable.

Discussion of Results

For the sake of completeness, the amplitude-sensitive stability boundary from Ref. 5 is reproduced here in Fig. 2. This boundary was obtained with a constant initial energy E_0 of 175, with $a_1(0)=-a_2(0)$ nonzero. Values of other system parameters are as shown in the caption. The complicated nature of the dependence of this stability boundary on the initial energy (and, ultimately, on $\mathbf{A}_0$) can be inferred from the shape of the boundary. The onset of instability over the whole boundary is characterized by the panel motion with time shown in Figs. 3 and 4. Figure 3 shows the motion of the panel at $x/a=0.75$, which is approximately where the peak displacement occurs. Figure 4 shows the mode shapes at fixed intervals of τ over a period or so. Note that the oscillation has a strong travelling-wave component. This behavior is quite similar to the "periodic but nonsimple-harmonic" behavior observed by Dowell[6] for large compressive values of R_x with linear aerodynamic loading. Here, apparently, for tensile as well as compressive inplane applied load, the mechanism of the instability involves motion of this sort. For values of R_x and λ on the unstable side of the linear stability boundary, the travelling-wave component subsides, and the motion resembles that for linear aerodynamic loading, with minor distortion.[5]

Figure 5 presents the variation of the critical value of λ, where instability first occurs, for both different levels of initial energy and different modal content. For $E_0=175$, for example, having the energy completely in the first mode ($a_1(0) \neq 0$) produces a critical value of λ slightly higher than that obtained with $a_1(0) = -a_2(0) \neq 0$. On the other hand, having all the initial

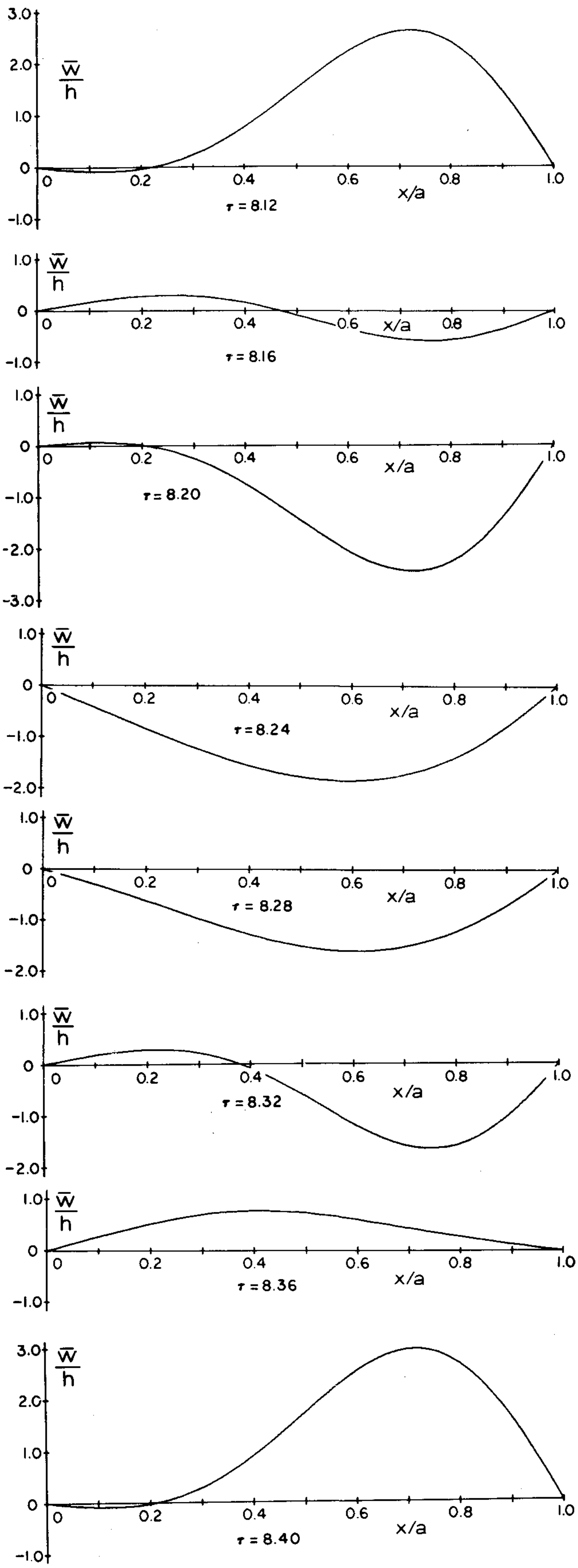

Fig. 4 Dimensionless panel displacement vs chordwise distance for panel of Fig. 3.

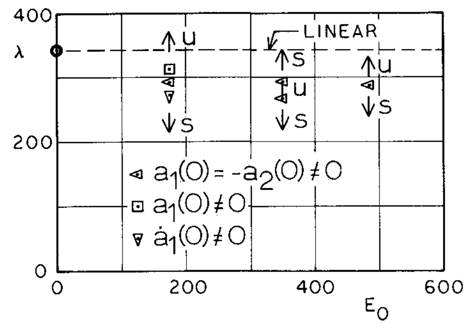

Fig. 5 Variation of critical value of λ with initial energy and modal content. $N = 6$, $R_x = 0$, $\mu/M = 0.01$, $Mh/a(\alpha)^{1/2} = 0.158$, $\Delta p = 0$.

energy as kinetic energy in the first assumed mode ($\dot{a}_1(0) \neq 0$) produces a slightly lower critical value of λ. With $a_1(0) = -a_2(0) \neq 0$ and $E_0 = 480$, the critical value of λ is only slightly lower than the value found with the same initial deflection shape and approximately one-third the energy. For $E_0 = 350$ and this same initial shape, the situation is more complicated. There is a band of values of λ for which the motion is unstable, bounded above and below by a stable region. This suggests a distorted version of the stability boundary of Fig. 2 near $R_x = -0.5\pi^2$, which for this value of E_0 encompasses the λ axis as well.

Figure 6 presents the effect of both positive and negative static pressure differences for $R_x = 0$ and the initial conditions of Fig. 2. Positive values of Δp, which tend to push the panel out of the cavity and into the freestream, are stabilizing, whereas negative values are destabilizing. This effect is related to the nature of the piston-theory aerodynamic term proportional to $(\partial w/\partial x)^2$. The sign on this term is such that a panel excursion in either transverse direction produces a pressure tending to push the panel into the cavity. This produces a strong asymmetry to the motion, as can be seen from Figs. 3 and 4. Negative values of Δp reinforce this term, while positive values act to counterbalance it.

The effect of varying the nonlinear interaction parameter $Mh/a(\alpha)^{1/2}$ is seen in Fig. 7. The initial conditions were fixed

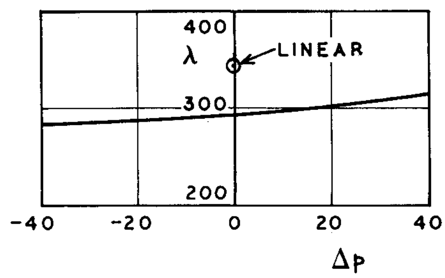

Fig. 6 Variation of critical value of λ with dimensionless static pressure difference for fixed initial conditions. $N = 6$, $R_x = 0$, $\mu/M = 0.01$, $Mh/a(\alpha)^{1/2} = 0.158$, $E_0 = 175$, $a_1(0) = -a_2(0) \neq 0$.

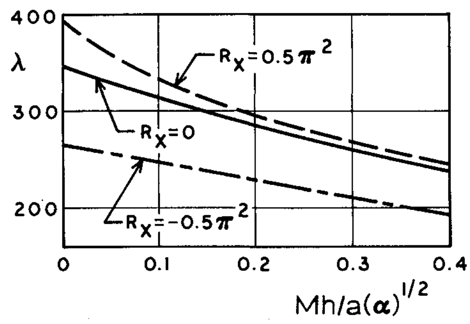

Fig. 7 Variation of critical value of λ with nonlinear interaction parameter for fixed initial energy. $N = 6$, $\mu/M = 0.01$, $\Delta p = 0$, $E_0 = 175$, $a_1(0) = -a_2(0) \neq 0$.

at those of Fig. 2, $E_0 = 175, a_1(0) = -a_2(0) \neq 0$. Increasing values of $Mh/a(\alpha)^{1/2}$, which represent increased relative importance of the nonlinear aerodynamic terms, produce monotonically decreasing values of the critical value of λ. A measure of the importance of the nonlinear aerodynamic effects can be obtained by examining the slope of these curves at the λ axis. This slope will be zero for zero initial energy and will in general decrease as the initial energy increases. The results from Fig. 5, however, suggest a complicated picture, and any trends observed must not be unduly generalized.

Conclusions

It has been shown theoretically that, under proper conditions, nonlinear aerodynamic loading can produce unstable panel motion in a parameter region that would be a stable one on the basis of a model with linear aerodynamics. The principal factors in determining the likelihood of such an instability are the excitation level that the panel is expected to encounter and the importance of the nonlinear aerodynamic loading in comparison with the stabilizing effect of in-plane stretching in the panel. The latter factor is measured directly by the interaction parameter $Mh/a(\alpha)^{1/2}$. For a given initial excitation, the critical value of λ varies quite smoothly with this parameter. On the other hand, the dependence of the instability on the nature of the initial conditions is quite complicated. This particular conclusion should not be too surprising, since the system itself is far from simple. It may well be that an inspired application of Lyapunov's method, or of some other approximate technique that deals directly with the continuous system, will ultimately provide more meaningful information.

To the best of the author's knowledge, amplitude-sensitive flutter as such has not been verified experimentally, at least for flat or slightly curved panels. Behavior of this sort is consistent with that noted by Ketter and Voss[7] in their experiments, where they found a hysteresislike variation of dynamic pressure for flutter with in-plane applied load. However, attempts to reproduce this behavior theoretically, with system parameters based on the experiments, were not successful. In fact, Ketter and Voss do offer another explanation for it, and the theoretical results (as shown in Fig. 2, for example) suggest that an amplitude-sensitive instability at the high tensile values of R_x involved—near 160—would be highly unlikely. Behavior suggestive of an amplitude-sensitive instability was also noted briefly by Anderson.[8] In his experiments, the freestream total pressure at which flutter stopped was slightly lower than that at which flutter began. In view of the rigidity of the panel edge mounting and the magnitude of the Mach number (2.81), it is unlikely that a nonlinear aerodynamic mechanism of the sort discussed here was very significant.

It should be noted that aerodynamic nonlinearity is not the only type of nonlinearity that can result in amplitude-sensitive instability. In Ref. 1, Dowell mentions some theoretical observations of instability to large disturbances when the in-plane compressive loading is greater than the still-air buckling load. These studies incorporated linear aerodynamic loading, and it is entirely possible that nonlinear aerodynamic loading could reinforce such behavior. In at least one example, as indicated in Fig. 2, this was not the case, but different initial conditions could produce very different results.

Since the type of panel considered here is far from a realistic one, no firm conclusions can be drawn regarding the practical impact of nonlinear hypersonic loading. However, it is clear that any assessment of this effect should include an accurate determination of the true in-plane restraint condition. A flat panel of finite span, which is also stabilized by stretching in the spanwise direction, should be less susceptible to nonlinear hypersonic effects in comparison with its two-dimensional counterpart. The introduction of curvature, particularly as far as shells are concerned, presents a different situation. It has been found[9] that a nonlinear structural shell model, with linear aerodynamic loading, exhibits a "softening" nonlinear behavior of the same type as is produced by nonlinear aerodynamic loading for the flat panels studied in this paper. The introduction of nonlinear aerodynamic loading into a shell analysis could very well reinforce significantly this behavior. Finally, there is the question of viscous aerodynamic effects. Within a linear framework, viscous effects on panel flutter are generally thought to be most important for Mach numbers near unity.[1] It is simply not known if this situation will change when nonlinear aerodynamic loading is significant.

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